UNIT 5 – ANGLES AND PARALLEL LINES

Assignment	Title	Work to complete	Complete
	Vocabulary	Complete the vocabulary words on the attached handout with information from the booklet or text.	
1	***Classifying Angles***	Classifying Angles	
	Activity	Five Angles	
2	***Referent Angles***	Referent Angles	
3	***Describing Angles***	Describing Angles	
4	***True Bearing***	True Bearing	
5	***Drawing a Right Angle Using a Compass and Ruler***	Drawing a Right Angle Using a Compass and Ruler	
6	***Copying An Angle Using a Compass and Ruler***	Copying An Angle Using a Compass and Ruler	
7	***Bisecting An Angle***	Bisecting An Angle	
8	***Parallel and Perpendicular Lines***	Parallel and Perpendicular Lines	
9	***Lines and Transversals***	Lines and Transversals	
10	***Parallel Lines and Transversals***	Parallel Lines and Transversals	
Mental Math	**Mental Math** Non-calculator practice	Get this page from your teacher	
Practice Test	**Practice Test** How are you doing?	Get this page from your teacher	
Self-Assessment	**Self-Assessment** ***"Traffic Lights"***	On the next page, complete the self-assessment assignment.	
Chapter Test	**Chapter Test** Show me your stuff!		

Traffic Lights

In the following chart, decide how confident you feel about each statement by sticking a red, yellow, or green dot in the box. Then discuss this with your teacher ***BEFORE*** you write the test.

Statement	**Dot**
After completing this chapter;	
• I can measure, and describe angles of various measures	
• I can draw to show the meaning the following types of angles: acute, right, obtuse, straight, reflex	
• I can draw and copy angles using a compass and ruler	
• I understand and can show the meaning of the terms complementary and supplementary angles	
• I can bisect angles using a protractor, or using a compass and ruler	
• I understand and can show the meaning of alternate interior angles, alternate exterior angles, corresponding angles, vertically opposite angles, interior and exterior angles on the same side of the transversal	
• I can determine whether or not lines are parallel using angles and a transversal	

Vocabulary: Unit 5

Angles and Parallel Lines

<table>
<tr>
<td>budget
*this term has been completed for you as an example</td>
<td><u>Definition</u>
an estimate of the amount of money to be spent on a specific project or over a given time frame</td>
<td><u>Diagram</u>: A sample of a personal monthly budget:
<table>
<tr><td>Net Pay</td><td></td><td></td><td>$2500</td></tr>
<tr><td>Rent</td><td>$600</td><td>Recreation</td><td>$100</td></tr>
<tr><td>Telephone</td><td>$75</td><td>Personal Care</td><td>$100</td></tr>
<tr><td>Utilities</td><td>$75</td><td>Savings</td><td>$150</td></tr>
<tr><td>Food</td><td>$500</td><td>Spending (CDs…)</td><td>$200</td></tr>
<tr><td>Transportation</td><td>$500</td><td>Other expenses</td><td>$100</td></tr>
<tr><td>Clothing</td><td>$100</td><td></td><td></td></tr>
<tr><td>Total</td><td></td><td></td><td>$2,500</td></tr>
</table>
</td>
</tr>
<tr>
<td>angle</td>
<td><u>Definition</u></td>
<td><u>Diagram/Example</u></td>
</tr>
<tr>
<td>angle bisector</td>
<td><u>Definition</u></td>
<td><u>Diagram/Example</u></td>
</tr>
<tr>
<td>angle measure</td>
<td><u>Definition</u></td>
<td><u>Diagram/Example</u></td>
</tr>
<tr>
<td>angle referent</td>
<td><u>Definition</u></td>
<td><u>Diagram/Example</u></td>
</tr>
</table>

acute angle	Definition An angle that has a measure bigger than 0^0 but smaller than 90^0	Diagram/Example
alternate exterior angles	Definition	Diagram/Example
alternate interior angles	Definition	Diagram/Example
complementary angles	Definition	Diagram/Example
corresponding angles	Definition	Diagram/Example

degree	Definition	Diagram/Example
obtuse angle	Definition An angle that has a measure greater than 90^0 but less than 180^0	Diagram/Example
parallel lines	Definition	Diagram/Example
perpendicular lines	Definition	Diagram/Example
reflex angle	Definition An angle that has a measure bigger than 180^0 but smaller than 360^0	Diagram/Example

right angle	Definition An angle that has a measure of exactly 90^0	Diagram/Example
straight angle	Definition An angle that has a measure of exactly 180^0	Diagram/Example
supplementary angles	Definition	Diagram/Example
transversal	Definition	Diagram/Example
vertically opposite angles	Definition	Diagram/Example

CLASSIFYING ANGLES

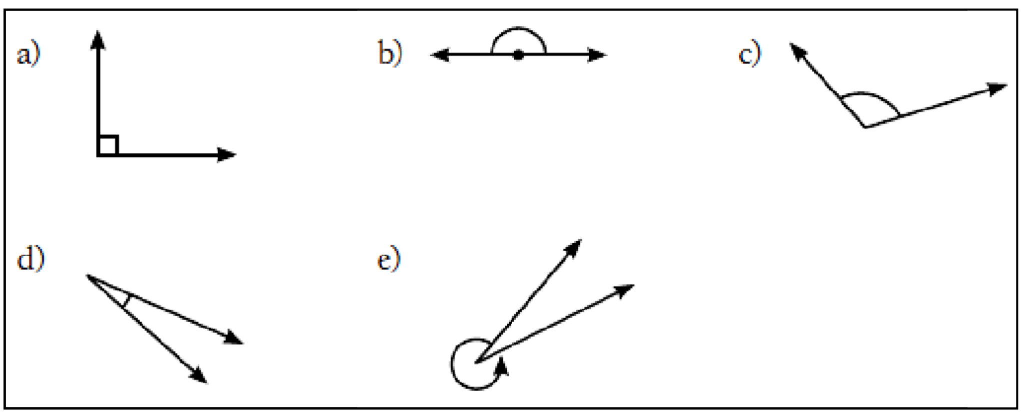
An **angle** is formed when two rays meet at a point called a **vertex**. For the purposes of this course, angles will always be measured in degrees, and can be measured with a protractor. There are 360^0 in a circle, and the angle measure you will be dealing with range from 0^0 to 360^0. Once the angle is formed, it is possible to classify it by how many degrees it is as follows:

Acute angle – measure is between 0^0 and 90^0
Right angle – measure is exactly 90^0; the two rays are perpendicular to each other
Obtuse angle – measure is between 90^0 and 180^0
Straight angle – measure is exactly 180^0
Reflex angle – measure is between 180^0 and 360^0

You will need to remember these terms and what they represent.

Example 1:

Identify the following angles.

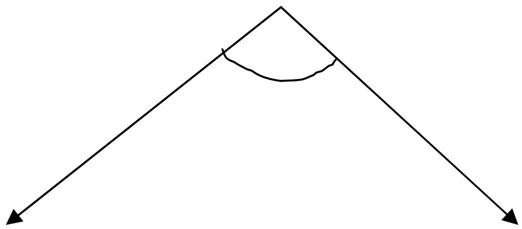


Solution:
a) This is a right angle. Notice the symbol between the rays to indicate perpendicular.
b) This is a straight angle.
c) This is an obtuse angle.
d) This is an acute angle.
e) This is a reflex angle. Notice the circle and arrow indicating which angle is of interest.

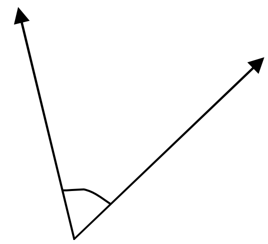
ASSIGNMENT 1 – CLASSIFYING ANGLES

1) Identify the type of angle indicated below.

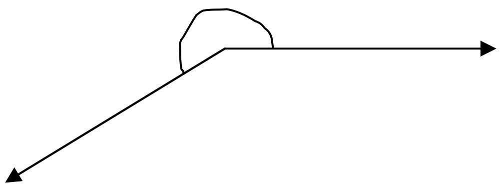
a)



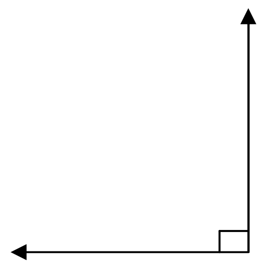
b)



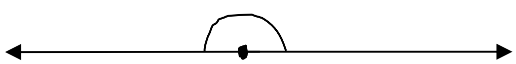
c)



d)



e)



f)

2) Identify the type of angle indicated.

a) 68^0

b) 215^0

c) 91^0

d) 32^0

d) 180^0

e) 89^0

f) 195^0

g) 265^0

h) 90^0

Activity 1 – Five Angles

Angle measures range from 0^0 to 360^0. In the attached chart, draw five angles of various measures, labelling the rays, vertices, and the angle. (The rays should be about 4 cm long.) These angles should illustrate the 5 different types of angles. (See below, #3). An example is done for you on the first line.

Now complete the chart for each of your angles:

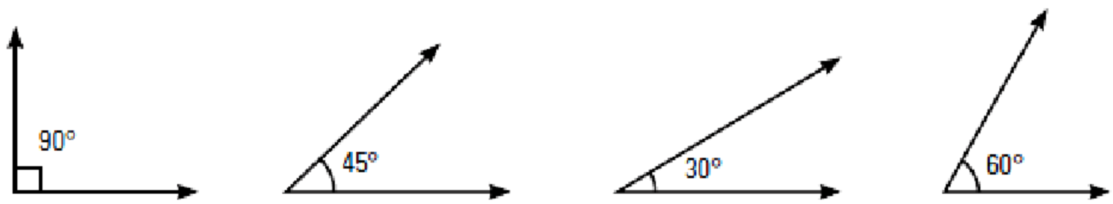
1. Carefully measure the angle you drew. You may need to extend the rays outside the box to do this accurately.
2. State the angle measure in degrees.
3. Identify the type of angle. The choices are acute, right, obtuse, straight or reflex. Explain why you chose each type.
4. Give an example of where you might see an angle of this type in the real world.

FIVE ANGLES				
	Angle	*Angle measure*	*Kind of angle (how do you know?)*	*Real-world example of this angle*
	Example 	20^0	acute: it's greater than 0^0 and less than 90^0	a stapler
1				
2				
3				
4				
5				

REFERENT ANGLES

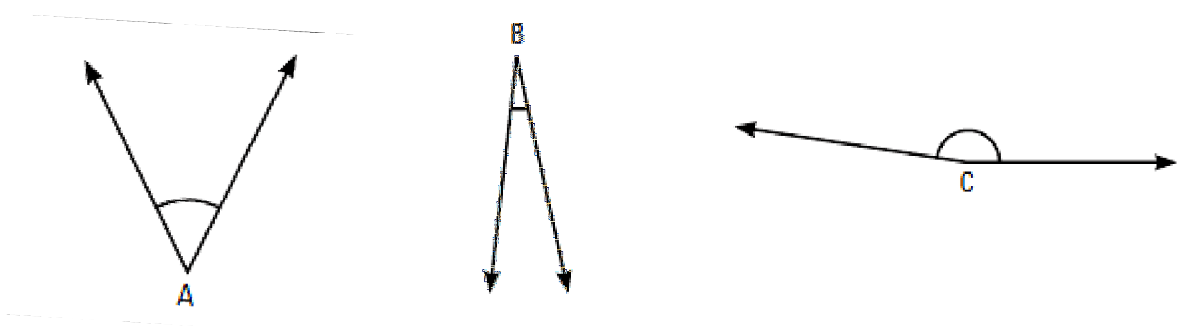
In many jobs, people have to draw angles or estimate their measures. To estimate the size of an angle, you can use referent angles. Referent angles are angles that are easy to visualize. You can use these referents to determine the approximate size of a given angle.

These are the referent angles commonly used:



When looking at the referents, these are the things to keep in mind. A right angle of 90^0 has the rays perpendicular to each other. A 45^0 angle is about half of a right angle. The 30^0 angle and the 60^0 angle are each smaller than and bigger than the 45^0 angle. These referents can be combined with each other, or with a straight angle to estimate larger angles.

Example: Use the referent angles above to estimate the size of these angles. After estimating their size, use a protractor to accurately measure each angle.



Solution:

Angle A (∠ A) is slightly bigger than the 45^0 referent angle so it is about 50^0.

Angle B (∠ B) is less than the 30^0 referent angle so it is about 20^0.

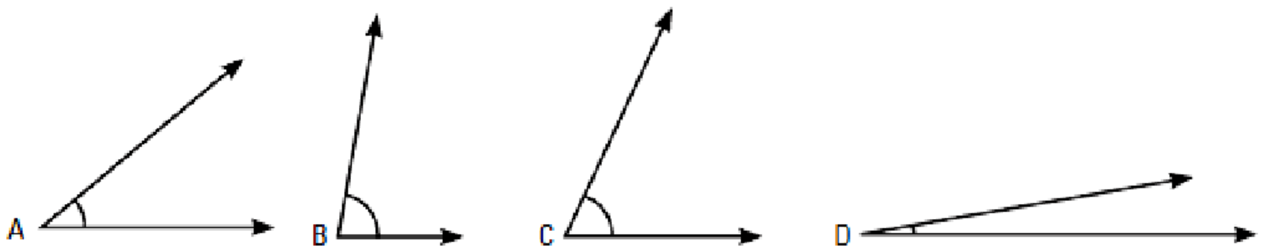
Angle C (∠ C) is slightly smaller than a straight angle (180^0) so it is about 170^0.

The symbol used to say “angle” is ∠ . So ∠ P reads as angle P.

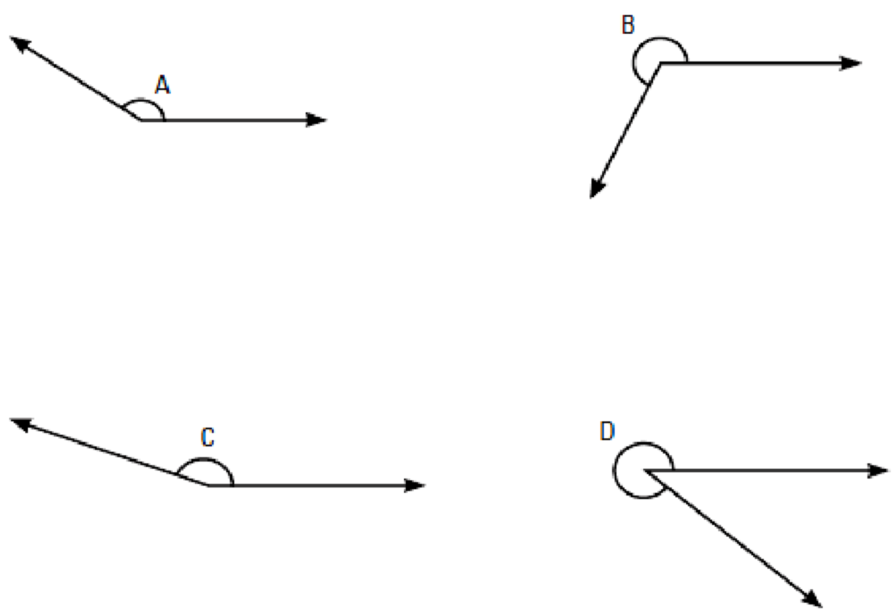
ASSIGNMENT 2 – REFERENT ANGLES

Use the referent angles to estimate the following angles.

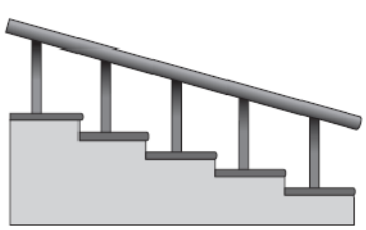
1)



2)



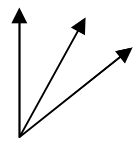
3) What is the approximate angle of the railing on the stairs below?



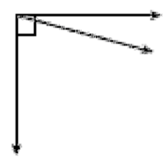
DESCRIBING ANGLES

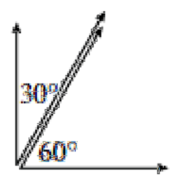
Angles are often described in pairs. There are three terms you will need to be familiar with in order to do this; adjacent angles, complementary angles, and supplementary angles.

Adjacent angles are angles that share a common vertex and a common ray as illustrated below.

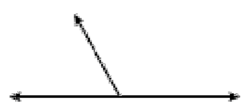


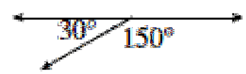
Complementary angles are two angles that when added together total 90^0 as shown below.





Supplementary angles are two angles that when added together total 180^0 as shown below.





Finding the complement and supplement of an angle can be done by subtraction. However, not all angles will have a complement or supplement depending on their size. If an angle is greater than 90^0, it will not have a complement, and if an angle is greater than 180^0, it will not have a complement or a supplement

Example: Find the complement and supplement for each angle, if they exist.

a) 75^0 b) 103^0 c) 300^0 d) 87^0

Solution: To find the complement, subtract the angle from 90^0. To find the supplement, subtract the angle from 180^0.

a) Complement: $90^0 - 75^0 = 15^0$ Supplement: $180^0 - 75^0 = 105^0$

b) Complement: does not exist Supplement: $180^0 - 103^0 = 77^0$

c) Complement: does not exist Supplement: does not exist

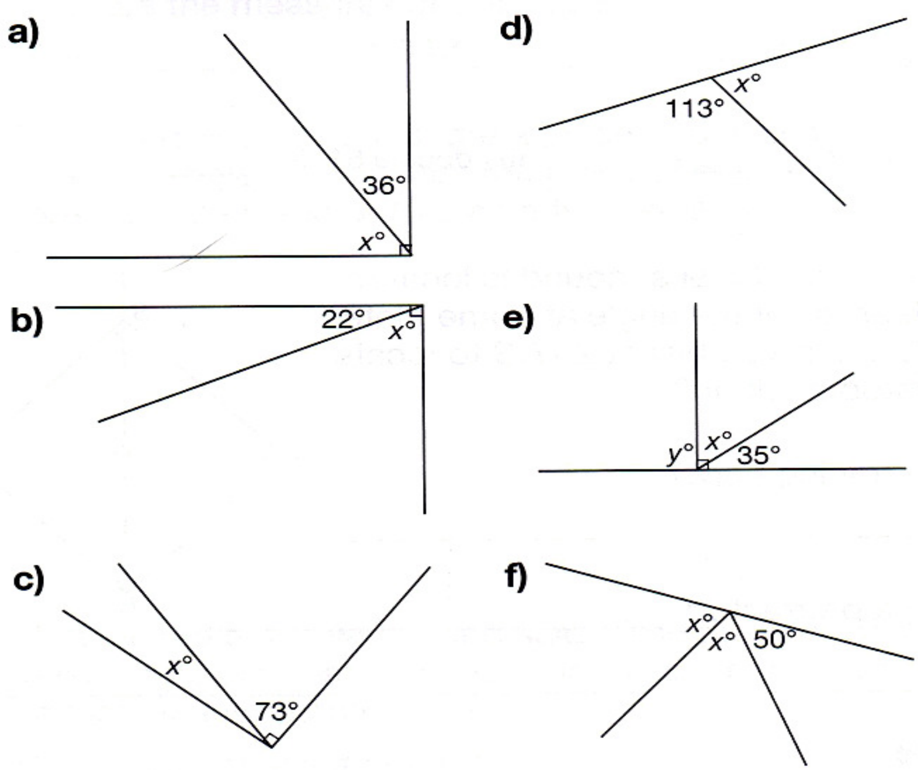
d) Complement: $90^0 - 87^0 = 3^0$ Supplement: $180^0 - 87^0 = 93^0$

ASSIGNMENT 3 – DESCRIBING ANGLES

1) Complete the following chart. If an angle does not exist, write “N/A”.

Angle	*Complement to angle*	*Supplement to angle*
53^0		
		121^0
	28^0	
		67^0
234^0		

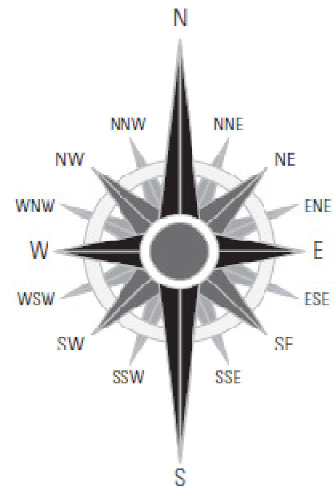
2) Determine the size of the *x* in each diagram below. Write the answer in the angle.



TRUE BEARING

In navigation and amp-making, people often measure angles from the vertical, or north. This angle, referred to as **true bearing**, is measured in a clockwise direction from a line pointing at north. Straight north has a bearing of 0^0.

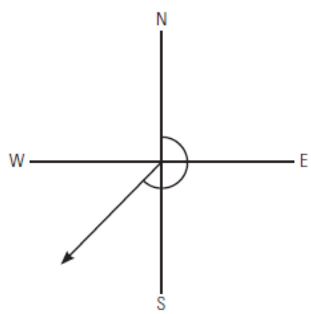
The compass rose is illustrated here. It shows many of the common bearings you might have heard of. You can see that east has a bearing of 90^0, south has a bearing of 18^0, and west has a bearing of 270^0. In between each of these cardinal directions, the bearings are half of the 90^0. So northeast (NE) has a bearing of 45^0 and SE has a bearing of $90^0 + 45^0 = 135^0$, and so on.



Each of the directions shown on this compass rose has a measure of 22.5^0. Through addition and subtraction, all other bearings can be found.

Example: What is the true bearing of a boat heading southwest?

Solution:



A boat heading southwest is 45^0 past south. So its bearing is:

$180^0 + 45^0 = 225^0$

ASSIGNMENT 4 – TRUE BEARING

1) A car is travelling 25^0 south of straight east. What is its true bearing? Show any calculations.

2) What is the true bearing of a boat travelling north-northwest? Show any calculations.

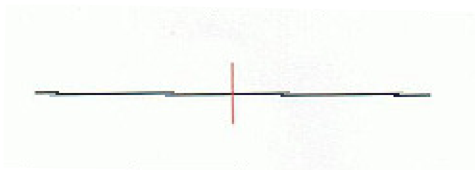
3) A boat is travelling WSW. Show 2 ways to calculate its true bearing.

DRAWING A RIGHT ANGLE USING A RULER AND COMPASS

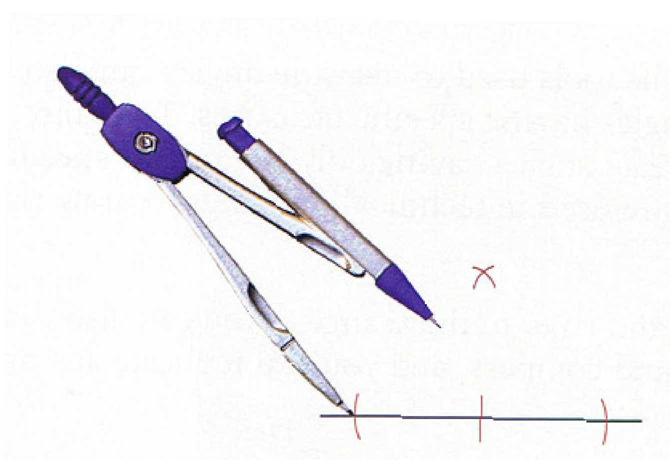
You have used a protractor to measure angles. You can also use the protractor to draw a 90^0 angle. Another way to draw a 90^0 angle is with a ruler and compass. This creates a perpendicular line to the original line given.

To create a right angle, follow these steps.

1) Draw a line segment and put a mark on it where you want the 90^0 angle to go.

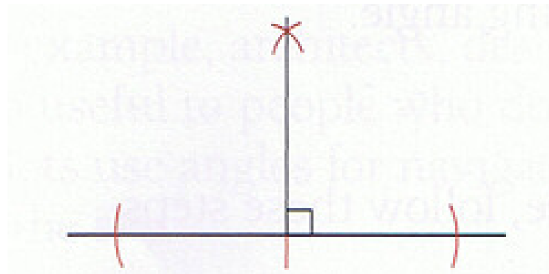


2) Put the compass point on the mark you made. Open the compass slightly and make two small marks (called arcs) on each side of the first mark along your line. Make sure you do not adjust the compass so the marks are the same distance on either side if the first mark.



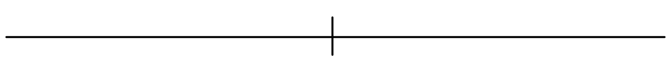
3) Open up the compass a bit more and then place the point on one of the new marks you just made. Make a small mark and then do the same thing after placing the compass point on the other new mark. Make sure these two new arcs cross each other.

4) Draw a line segment that goes through the point where the arcs crossed and the first mark you made. The new line and the first line you drew are perpendicular to each other, and therefore form a 90^0 or right angle.



<u>ASSIGNMENT 5 – DRAWING A RIGHT ANGLE USING A RULER AND COMPASS</u>

1) Draw a perpendicular line to the line on the page, using only a compass and a ruler. Ask your teacher for a compass if you don't have one. Do not erase any of your construction marks from the compass.

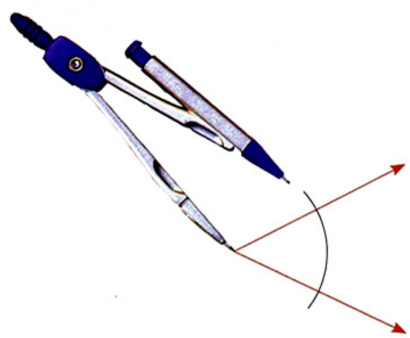


COPYING AN ANGLE USING A COMPASS AND RULER

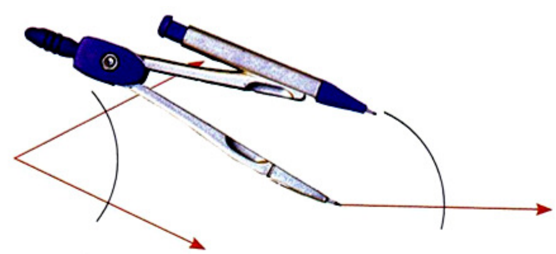
You can also copy any angle with a compass and ruler. This is also referred to as replicating the angle, and is useful when you want to copy an angle from one figure to another out measuring.

To copy an angle, follow these steps.

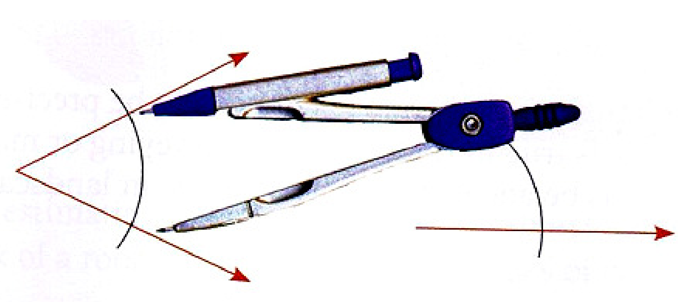
1) Start with the angle you want to copy on your page. Using your compass, put the point of the compass on the vertex of the angle and draw an arc across both of the legs of the original angle.



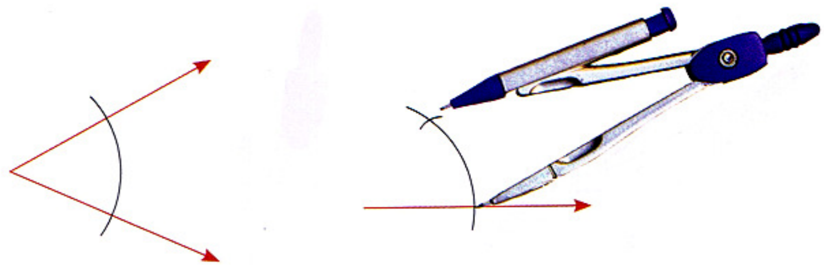
2) Using a ruler, draw one leg of your new angle somewhere else on the page. Without adjusting the compass, put the point of the compass on one end of your new line and draw an arc of the same length as the one you drew on the original angle.



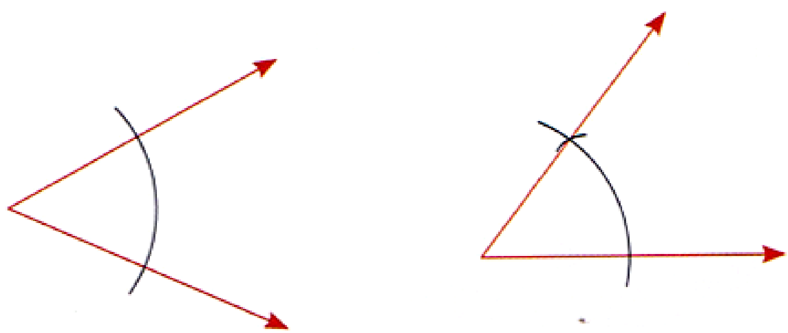
3) Bring the compass up to the original angle, and set it so that its point and pencil tip touch points where the original arc intersects the sides of the angle. This measures the size of the angle using the compass.



4) On the new angle, place the compass point on the point where the side and the new arc meet. Draw a short arc through the big arc you drew before.

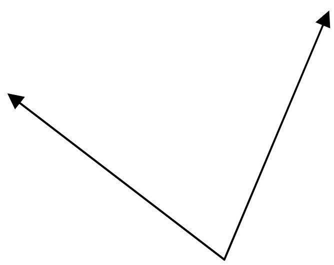


5) Use the ruler to draw the other side of the angle, from the vertex through the point where the two arcs intersect. The result is a new angle with the same measurement as the original angle.



ASSIGNMENT 6 – COPYING AN ANGLE USING A COMPASS AND RULER

1) Copy the angle to another location on this page using only a compass and ruler. Ask your teacher for a compass if you don't have one. Do not erase any of your construction marks from the compass.

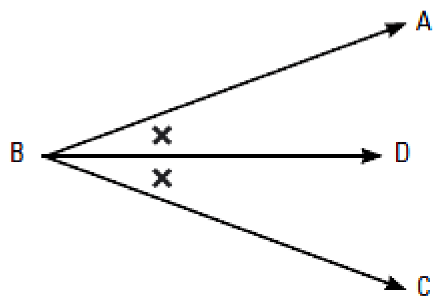


BISECTING AN ANGLE

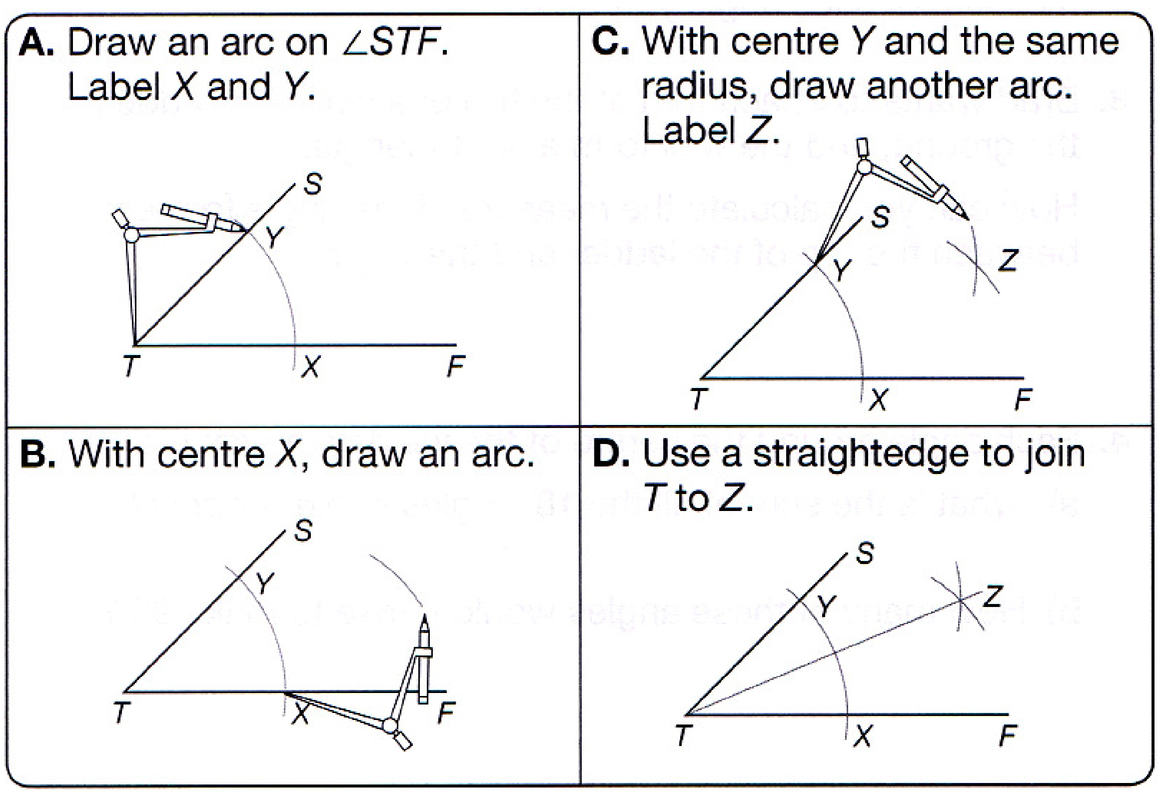
To bisect something means to cut it into two equal parts. An angle can be bisected into another ray, splitting the angle into two smaller but equal angles.

There are 2 ways to bisect angles and straight lines:

1) Using a protractor, measure the angle. Divide that measure in two and measure the resulting angle within the original angle.

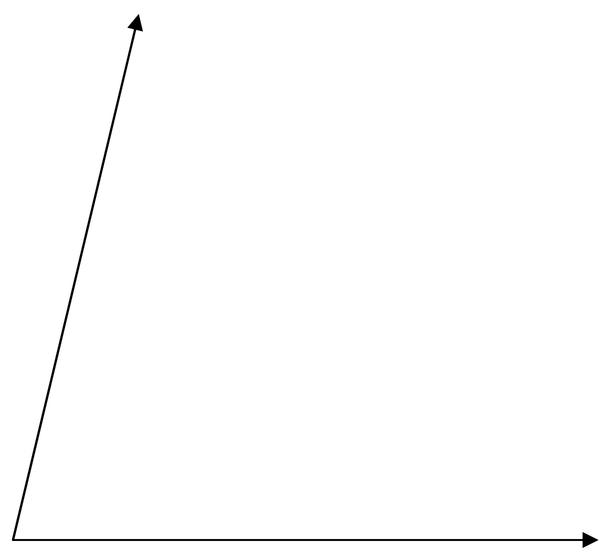


2) To bisect an angle without a protractor, use a compass and a ruler. Complete the following steps:



ASSIGNMENT 7 – BISECTING AN ANGLE

1) Bisect the angle below using only a compass, ruler, and pencil. Ask your teacher for a compass if you don't have one. Do not erase any of your construction marks from the compass.

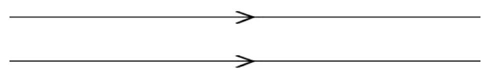


2) If a right angle is bisected, what is the size of each angle?

3) An angle is bisected and the resulting angles are 78^0 each. How big was the original angle?

PARALLEL AND PERPENDICULAR LINES

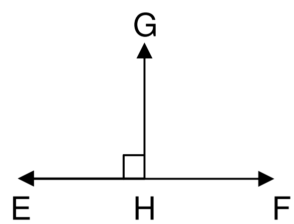
Parallel lines are lines that are always the same distance apart. Parallel lines will *never* cross. A short form way of indication that lines are parallel is to use the symbol $\parallel$. So if you read the following: AB $\parallel$ CD, it would mean that line AB is parallel to line CD. Arrows on the lines are also used to show that the lines are parallel.



We have already discussed perpendicular line. **Perpendicular lines** are two lines that meet at a right angle – 90^0. The symbol to show that two lines are perpendicular when writing about them is $\perp$. So to say that line EF is perpendicular to line GH you might see the following:

$$EF \perp GH$$

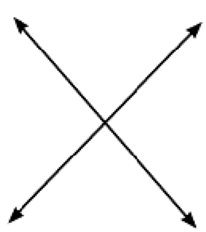
The little box in the corner of a right angle also illustrates this.



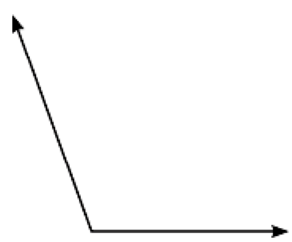
ASSIGNMENT 8 – PARALLEL AND PERPENDICULAR LINES

1) Using a protractor, determine whether these lines are perpendicular.

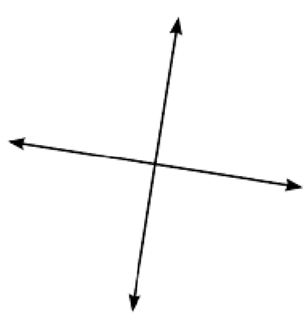
a)



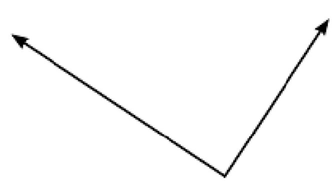
b)



c)

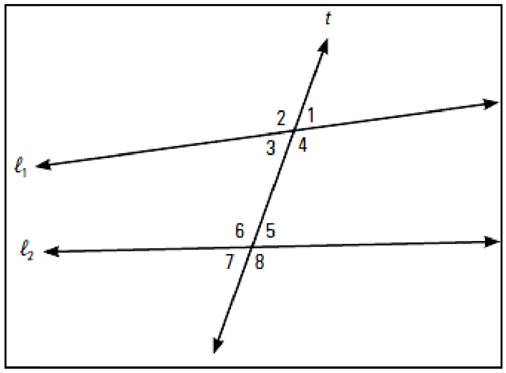


d)



LINES AND TRANSVERSALS

Many angles can be formed by two lines and a transversal. By definition, a **transversal** is just a line that intersects (or crosses) two or more other lines. Whether or not the two lines the transversal crosses are parallel, there are specific relationships formed when lines intersect. The illustration below shows these relationships.



In this diagram, the two lines, ℓ_1 and ℓ_2 are clearly not parallel. The relationships described below hold true for parallel lines too.

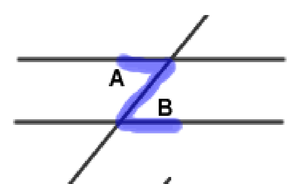
vertically opposite angles: angles that are created by intersecting lines, these angles are opposite each other

Examples: $\angle 1$ and $\angle 3$ $\angle 2$ and $\angle 4$ $\angle 5$ and $\angle 7$ $\angle 6$ and $\angle 8$

interior alternate angles: angles in opposite positions between two lines intersected by a transversal and also on alternate sides of the same transversal. These angles can be thought of as a "Z" pattern – see the illustration to the right. Note that the "Z" can be stretched out or backwards too.

Examples: $\angle 3$ and $\angle 5$ $\angle 4$ and $\angle 6$

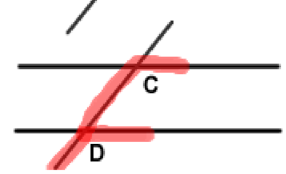
Two alternate interior angles form a Z shape: Z, N, Σ, or И.



corresponding angles: two angles that occupy the same relative position at two different intersections. These angles can be thought of as an "F" pattern – see the illustration to the right. Note that the "F" can be upside down or backwards too.

Examples: $\angle 1$ and $\angle 5$ $\angle 2$ and $\angle 6$

$\angle 3$ and $\angle 7$ $\angle 4$ and $\angle 8$



Two corresponding angles form an F shape: F, ш, ⅎ, or т.

exterior alternate angles: angles in opposite positions outside two lines intersected by a transversal and also on alternate sides of the same transversal. These angles can be thought of as an outside "Z" pattern – see the illustration to the right. Note that the "Z" can be stretched out or backwards too.

Examples: ∠ 1 and ∠ 7 ∠ 2 and ∠ 8

interior angles on the same side of the transversal: angles inside the two lines that are intersected by the transversal and also on the same side of the transversal. This forms a "C" pattern.

Examples: ∠ 3 and ∠ 6 ∠ 4 and ∠ 5

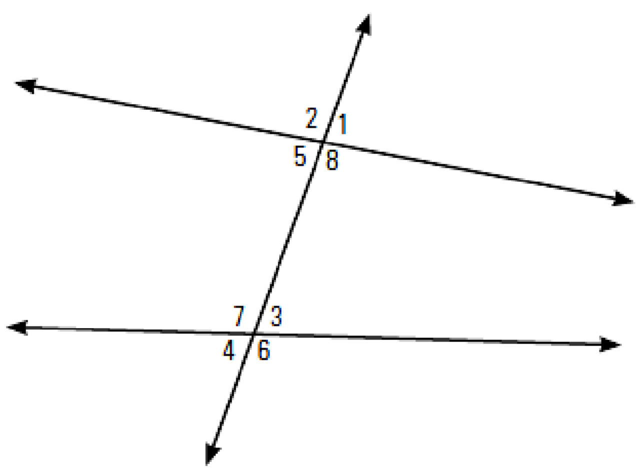
> Two interior angles on the same side of a transversal form a C pattern: ⊏, ⊔, ⊐, or ⊓.

exterior angles on the same side of the transversal: angles outside the two lines that are intersected by the transversal and also on the same side of the transversal.

Examples: ∠ 1 and ∠ 8 ∠ 2 and ∠ 7

ASSIGNMENT 9 – LINES AND TRANSVERSALS

1) Using the diagram below, identify the relationship between each pair of angles named.



a) ∠ 7 and ∠ 8 ____

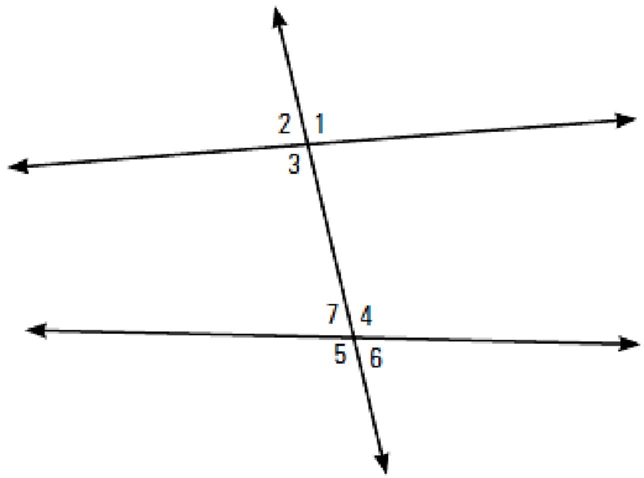
b) ∠ 2 and ∠ 7 ____

c) ∠ 1 and ∠ 6 ____

d) ∠ 5 and ∠ 7 ____

e) ∠ 6 and ∠ 7 ____

2) Using the diagram below, identify the following angles.



a) an exterior alternate angle to ∠ 2

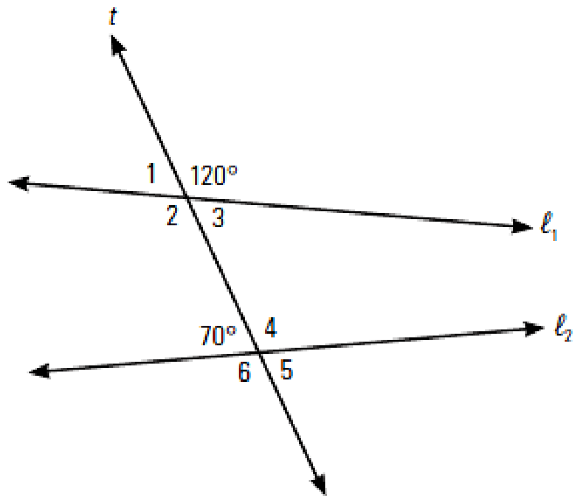
b) an interior angle on the same side of the transversal as ∠ 7

c) an interior alternate angle to ∠ 4

d) a corresponding angle to ∠ 5

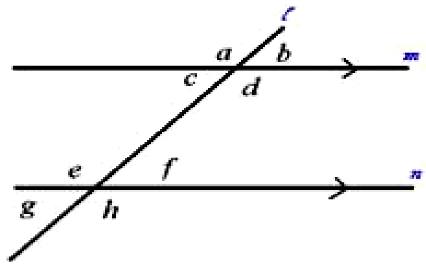
e) a vertically opposite angle to ∠ 3

3) Using the diagram below, calculate the size of each unknown angle indicated in the figure. Hint: Remember what the angle measure of a straight angle is. Show your calculations below the diagram.



PARALLEL LINES AND TRANSVERSALS

So far, the transversals we have used have not crossed parallel lines. If the lines are parallel however, there is more information about the measures of the angles that can be determined.

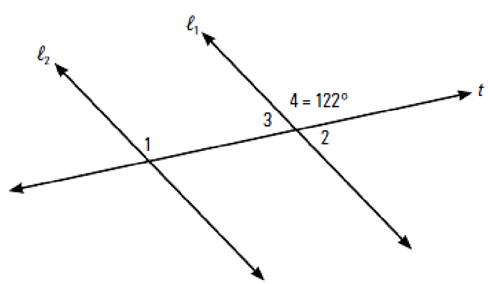


If the lines crossed by the transversal are parallel, the following are true:

- the interior alternate angles are equal
- the exterior alternate angles are equal
- the corresponding angles are equal
- interior angles on the same side of the transversal are supplementary
- exterior angles on the same side of the transversal are supplementary

Conversely, if you know that two lines are crossed by a transversal and any of the above relationships are true, then the angles must be parallel.

Example: In the diagram below, the two lines, ℓ_1 and ℓ_2 are parallel. What are the measures of the three angles indicated? Explain your answers.



Solution:

$\angle 1 = 122^0$ – it is a corresponding angle to $\angle 4$ so they are equal.

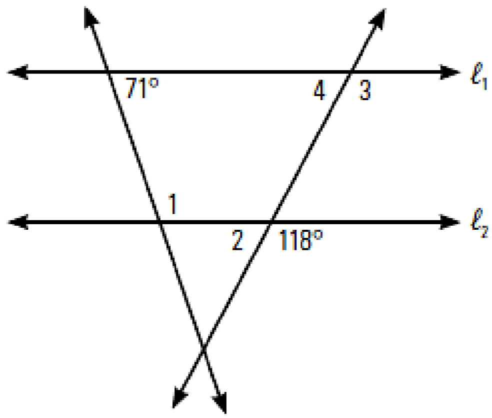
$\angle 2 = 58^0$ – it is the supplement to $\angle 4$ so they add up to 180^0.

$\angle 3 = 58^0$ – it is vertically opposite to $\angle 2$ so they are equal.

NOTE: This is not the only way to determine the angle measures. There are other possible solutions to find the same answers.

ASSIGNMENT 10 – PARALLEL LINES AND TRANSVERSALS

1) In the diagram below, the two lines, ℓ_1 and ℓ_2 are parallel. What are the measures of the three angles indicated? Explain your answers.



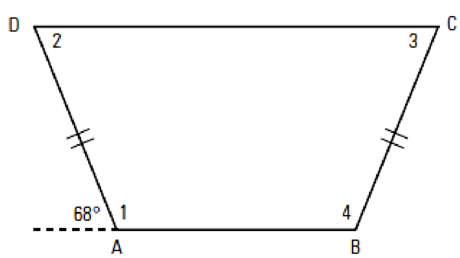
∠ 1 ____

∠ 2 ____

∠ 3 ____

∠ 4 ____

2) A trapezoid is a special quadrilateral (4-sided shape) that has one set of opposite sides parallel, and the other side not parallel. What are the measures of the trapezoid shown below? Hint: Be careful of the order in which you calculate the angles. The double ticks on the sides mean that those sides are equal in length.



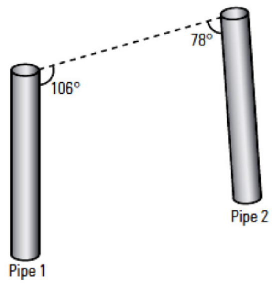
∠ 1 ____

∠ 2 ____

∠ 3 ____

∠ 4 ____

3) The diagram below shows two pipes that are vertical but not yet parallel to each other. How much must the second pipe be moved (what angle) to make them parallel? Show your calculations.



4) In the diagram below, the side of the house and the side of the attached shed are parallel. What are the measures of ∠ 1 and ∠ 2?

5) A plumber must install pipe 2 parallel to pipe 1. He knows that ∠ 1 is 53^0. What is the size of angle ∠ 2?